

# Fitting Tidal Constituents to Altimeter Data

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**Abstract**—The gravitational influence of the Moon and Sun have a well-known influence on the Earth’s ocean levels: tides. Radar altimeters make precise measurements of the height of the instrument above ground. With a known altimeter orbit and measured altitude, the height of the ocean tide can be measured to centimetric scales. Accounting for the tidal forces from celestial objects is an important step in finding the residual sea level height.

In this paper radar altimeter data from Jason-2 is used to estimate the magnitude and phase of eight dominant tidal constituents over six study regions. Each constituent is estimated by fitting the data to a Fourier series to estimate tidal magnitude and phase. Accounting for the tidal components selected for this paper largely models the tides, but some residual variance remains. This is attributed to an incomplete tidal model and to measurement and model error.

## I. INTRODUCTION

DAILY changes in ocean levels have long been observed by people dwelling near coasts. There is evidence that ancient civilizations noted the correlation between phases of the moon and the occurrence of tides. Beginning in the 17th century, Isaac Newton and others began to successfully explain tides in terms of the change in gravitational influence of the Earth, Moon, and Sun as they move around each other. While this explains a large portion of the tides, other factors, including the Earth’s rotation and ocean floor depth, also play a role.

Traditionally, measurements of tides are conducted near harbors, piers, or other objects close to the shoreline. These objects provide a frame of reference against which to measure the change in ocean level. While this provides a good sampling of tides near coasts, it is more difficult to measure the tides over the open ocean. If a ship is in the middle of the ocean, how are tides measured if there’s nothing to measure against? One solution is to use a spaceborne altimeter.

An altimeter is a type of radar that measures the height of the instrument above the ground. Because electromagnetic waves have a known propagation speed ( $c$ , the speed of light), the time delay of an echo from a transmitted pulse can be directly mapped to distance, or height. In order to determine the height of the ocean and not merely the height of the altimeter, a reference level must be used. In this case the reference level is defined as the reference ellipsoid, a surface that approximates the geoid, the gravitational equipotential earth surface. The altitude above the reference ellipsoid is determined by using calculated ephemerides (orbital elements) of the orbiting satellite hosting the altimeter. Ocean level is then defined as the range from the altimeter to the ocean surface subtracted from the altitude of the altimeter (“range” to the reference ellipsoid).

By globally monitoring the ocean levels using Earth-orbiting altimeter data, tide levels in the open ocean can be measured. This paper does not empirically derive the tidal constituents but

instead estimates the amplitude and phase of eight known major tidal constituents. This is done using harmonic analysis, or fitting a Fourier series to the data in the least-squares sense. The periods of the tidal components are known, but the amplitude and phase of each component are estimated based on the Fourier series fit.

The altimeter chosen for the paper is Jason-2, the modern successor to previous successful spaceborne altimeters Topex/Poseidon and Jason-1. Further background to Jason-2, altimetry, and tides are first presented. The methodology and study results follow.

## II. BACKGROUND

Jason-2 is first introduced. A short background to radar altimetry follows. Ocean tides and tidal aliasing are presented. Finally, the Jason-2 data format is discussed.

### A. Jason-2

The data used in this study is from the OSTM/Jason-2 mission, launched in June 2008. It is the third generation in a successful series of radar altimeters. The first, Topex/Poseidon (T/P), collected data from 1992 to 2005. The second, Jason-1, was launched in 2001 and is currently operating in tandem with Jason-2. To maintain a consistent data set, many of the features of all three missions are similar, such as operating frequencies and satellite ground track.

The altimeter hosted on Jason-2 is Poseidon-3. It operates at both Ku-band (13.575 GHz) and C-band (5.3 GHz), as with Jason-1 and T/P. Also onboard is a microwave radiometer used to measure atmospheric delays induced by the presence of water vapor [1]. The altimeter is nadir-pointing and has a circular ground footprint size on the order of 30 km in diameter. The vertical resolution of Jason-2 is better than 3.4 cm.

### B. Radar altimetry

At the most fundamental level, radar altimetry is based on determining the range to a target ( $R$ ), which is a function of the travel time of an electromagnetic wave ( $t$ ) and the speed of propagation in the medium ( $c$ ). This can be expressed as

$$R = ct/2, \quad (1)$$

taking into account the two-way travel time for a wave from the radar to the target and back again. The various details of altimetry are based upon modifications to Eqn. (1). A comprehensive examination of altimetry may be found in [2], but such details are beyond the scope of this paper.

The quantity most useful for oceanography is not the range  $R$  of the instrument to the ocean surface, but the height  $h$  of the ocean above or below some reference level. With  $H$  taken

to be the height of the altimeter above a reference geoid, the ocean height  $h$  is then

$$h = H - R. \quad (2)$$

A geoid is an equipotential surface. If sea level were undisturbed by waves, winds, and land, the mean sea level would be the geoid.

For a spaceborne radar altimeter,  $R$  is found by Eqn. (1) and  $H$  is determined by measuring the orbit of the platform. On Jason-2, a GPS receiver and other instruments are used to determine the orbit parameters to high precision.

Measurements of  $R$  are affected by atmospheric attenuation. Both scattering and absorption in dry-air conditions attenuate the radar signal. The effects of water vapor in the atmosphere further attenuate the signal. Additional water in the form of clouds and rain even further attenuates the radar signal [2]. Atmospheric water content is estimated on Jason-2 using a three-frequency radiometer. This estimate is used to correct the range measurement. The attenuation and refraction of the atmosphere and other effects on Eqn. (1) are accounted for in the design of Jason-2.

### C. Ocean tides

The primary forces driving ocean tides are gravitational influences from the Moon and the Sun. A simple model of the Earth-Sun-Moon system explains the primary tidal components: a semi-diurnal tide corresponding to the position of the Moon, and another semi-diurnal tide corresponding to the position of the Sun. Harmonics for these tides exist, as well as tidal components due to the elliptic and inclined nature of the orbits and other parameters. For the purposes of this paper, only the eight strongest tidal constituents are estimated for various regions.

### D. Tidal aliasing

According to the well-known Nyquist-Shannon sampling theorem, any frequency content in a regularly sampled signal that is above half the sampling frequency is aliased to lower frequencies. In referring to tidal components, the period of the signal is used more than its frequency. Restating the sampling theorem in this case, any content with a period less than twice the sample period will be aliased to a longer period.

For Jason-2, the sample period (orbit repeat period) is  $\Delta t = 9.9156$  days. Measurements of a tidal harmonic at the same location but separated in time have an alias period of

$$T_a = \frac{2\pi\Delta t}{2\pi(f\Delta t - [f\Delta t + 0.5])}, \quad (3)$$

where  $f$  is the tide component frequency,  $\Delta t$  is the sample period, and  $[\cdot]$  is the `fix` function, which returns the greatest integer less than the argument. Table I lists the tidal constituents considered in this paper and their periods and aliased periods [3], [4]. While the eight tidal constituents (and other constituents not considered in this paper) in Tab. I alias to longer periods, the orbit of Jason-2 is designed such that the dominant tidal constituents do not alias to periods associated with other geophysical phenomena.

TABLE I  
TIDAL CONSTITUENTS, THEIR PERIODS  $T$ , AND THE ALIASED PERIODS  $T_a$   
UNDER JASON-2 SAMPLING [4].

Tide	$T$ (hours)	$T_a$ (days)	Tide description
$M_2$	12.42	62.11	Principal lunar semidiurnal
$S_2$	12.00	58.74	Principal solar semidiurnal
$N_2$	12.66	49.53	Larger lunar elliptic semidiurnal
$K_2$	11.97	86.60	Luni-solar declinational semidiurnal
$K_1$	23.93	173.19	Luni-solar declinational diurnal
$O_1$	25.82	45.71	Lunar declinational diurnal
$P_1$	24.07	88.89	Principal solar
$Q_1$	26.87	69.36	Major lunar elliptical

TABLE II  
REGIONS SELECTED FOR THE STUDY. A REGION OR TWO IS SELECTED  
FROM SEVERAL DIFFERENT OCEANS.

Number	Longitude range	Latitude range	Region description
1	110 W–111 W	20 S–19 S	South Pacific
2	145 W–146 W	15 N–16 N	North Pacific
3	75 E–76 E	1 S–0 S	Indian
4	20 W–21 W	41 S–40 S	South Atlantic
5	50 W–51 W	25 N–26 N	North Atlantic
6	48 W–49 W	10 N–11 N	North Atlantic

When the samples of the tidal harmonic are at different times and at different locations, then the phase of the tide is also aliased because the tide is not stationary. This is ideally accounted for but is not included in this paper. As a consequence, only regions with a  $\Delta$  longitude smaller than the Jason-2 track spacing should be used in the method below. The Jason-2 track spacing is  $\Delta x = 2.835^\circ$  [3].

### E. Jason-2 data format

The Jason-2 data is mirrored in several locations online. This paper uses the final version of the GDR data (geophysical data record) from the National Oceanographic Data Center (NODC), located at `ftp://data.nodc.noaa.gov/pub/data/nodc/jason2/gdr/gdr`. The data is stored in the NetCDF file format, a self-describing platform-independent data format commonly used for scientific data. Each file contains data from one pass or rev, nearly an hour's worth of measurements.

## III. METHOD

In order to estimate tide component amplitudes and phases, small regions are selected to average the data over. As shown in Tab. I, some tidal components have long aliased periods, so Jason-2 data is collected over about 130 days. As discussed above, phase aliasing prohibits a spatial region larger than about  $2.8^\circ$  longitude, so the regions are limited accordingly. Each region is defined to be a  $1^\circ$  by  $1^\circ$  box.

The land-flagged data is masked out since those samples are not relevant to ocean tide estimation. Also, altimeter data that is flagged as being poor quality is discarded. Several regions are defined over some of the Earth's oceans, as listed in Tab. II. The regions chosen are over the open ocean, since shallow waters near coastlines greatly complicate tidal estimation (e.g.,

wave reflections from land and ocean bottom need to be taken into account).

For each Jason-2 pass over each small region, all ocean height measurements are collected and averaged. The mean value for the pass is stored. This results in a time series of ocean height for each region spanning the 130-day length.

To determine ocean height, several variables are collected from the data files. First, the sea surface height is determined by Eqn. (2), which is expressed in data variable names:

$$\text{Sea Surface Height} = \text{Altitude} - \text{Corrected Range}. \quad (4)$$

The altitude is directly measured and is the height of Jason-2 above the reference ellipsoid. The corrected range is the range to the sea surface that also accounts for absorption and scattering from the atmosphere [1]. In order to account for other effects, the residual sea height for the purposes of this paper is defined as

$$\begin{aligned} \text{Residual Sea Height} = & \text{Sea Surface Height} \\ & - \text{Mean Sea Surface} \\ & - \text{Solid Earth Tide Height} - \text{Pole Tide Height} \\ & - \text{Inverted Barometer Height Correction} \\ & - \text{High Freq. Sea Surface Topography Fluctuations}. \end{aligned} \quad (5)$$

All of these values are given in the Jason-2 data files. A description of each variable is beyond the scope of the paper, but the residual sea height represents the sea height after accounting for many effects that are not related to ocean tide estimation [1].

After collecting the time series of residual sea surface height for each region, the tidal components are estimated. This is performed by creating a first-order Fourier series for each of the eight tidal components. Each Fourier series uses the aliased tide period from Tab. I. The amplitude and phase for each component is estimated using a least-squared fit to the irregularly sampled data. Summing the eight Fourier series fits creates the total eight-tide fit. This is represented as

$$x(t) = \sum_{k=1}^8 D_k \cos(\omega_k t + \theta_k), \quad (6)$$

where  $D_k$  and  $\theta_k$  are the magnitude and phase of tide component  $k$  with angular frequency  $\omega_k = 2\pi/T_k$  and  $x(t)$  is the fit as a function of time.

#### IV. RESULTS

Figures 1–6 show the residual sea surface height as defined by Eqn. (5) (with mean removed), as well as the fit based on estimating the amplitude and phase of each of the eight tidal constituents for each of the six regions. Each figure caption also lists the  $\ell_2$  norm of the error  $\|\mathbf{x} - \hat{\mathbf{x}}\|$ , where  $\mathbf{x}$  is the measured data and  $\hat{\mathbf{x}}$  is the eight-tide fit. Table III lists the amplitude and phase for each of the eight tide components for each of the six study regions.

The tidal fits largely describe the data well. However, there is still some variability around the tide fit. This is attributed to an incomplete tidal model as well as other residual surface

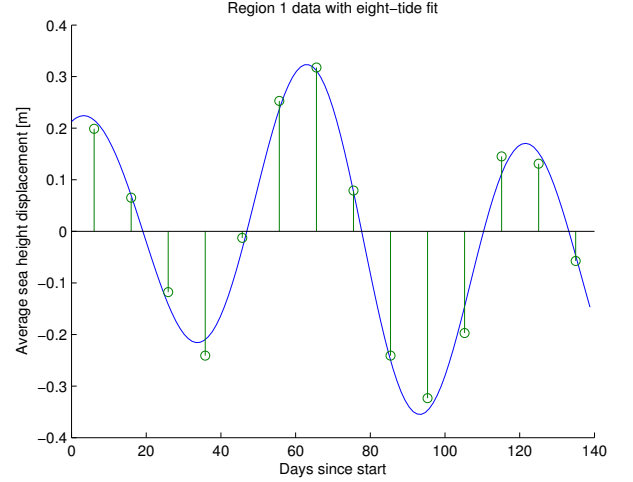


Fig. 1. Residual sea surface height for region 1 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.087 m.

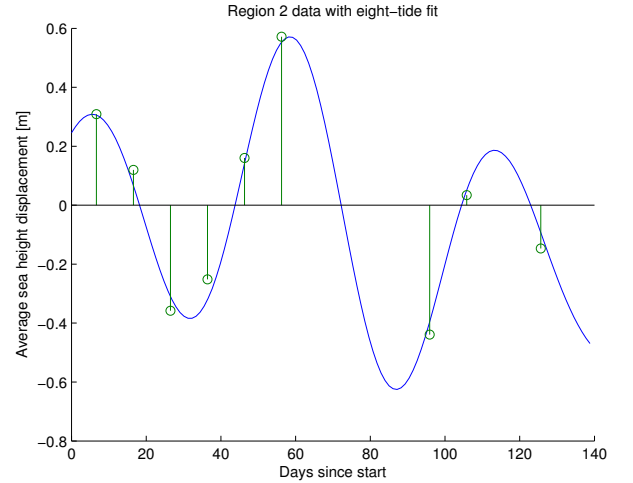


Fig. 2. Residual sea surface height for region 2 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.125 m.

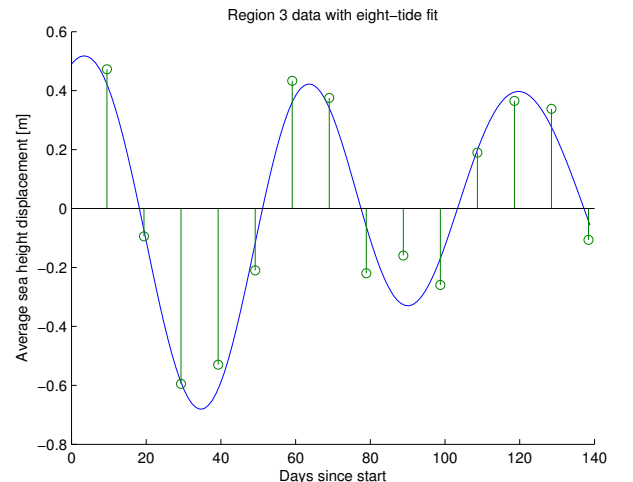


Fig. 3. Residual sea surface height for region 3 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.308 m.

TABLE III  
THE ESTIMATED TIDAL CONSTITUENTS FOR EACH REGION. THE MAGNITUDE IS IN METERS, THE PHASE IS IN DEGREES.  
FOR EACH REGION, THE TIDES ARE ORDERED FROM STRONGEST TO WEAKEST.

Region	Tide	Magnitude	Phase	Region	Tide	Magnitude	Phase
1	M2	0.2528	2.48	4	M2	0.2175	-148.86
	K1	0.0876	87.90		K1	0.0633	-179.31
	O1	0.0459	169.58		O1	0.0561	-74.17
	S2	0.0315	22.60		S2	0.0282	140.73
	P1	0.0159	51.16		P1	0.0248	-152.22
	Q1	0.0078	172.65		N2	0.0150	69.99
	N2	0.0073	-72.21		Q1	0.0060	-116.77
	K2	0.0022	69.62		K2	0.0039	-157.66
2	M2	0.3329	-14.11	5	M2	0.1634	-101.09
	K1	0.1959	92.5		K1	0.0674	-17.41
	O1	0.1750	118.66		O1	0.0291	10.88
	S2	0.0961	-7.94		S2	0.0203	-121.83
	P1	0.0507	81.78		P1	0.0201	173.15
	N2	0.0373	-43.04		N2	0.0072	161.38
	K2	0.0218	101.51		Q1	0.0060	31.77
	Q1	0.0135	117.51		K2	0.0016	-31.67
3	M2	0.4407	4.42	6	M2	0.5828	-95.73
	P1	0.1356	26.37		K1	0.1741	139.07
	O1	0.1223	115.61		O1	0.1193	-59.04
	K1	0.0717	-120.8		P1	0.0573	-131.79
	N2	0.0349	-104.7		S2	0.0415	94.67
	S2	0.0346	-3.52		N2	0.0294	66.91
	K2	0.0085	30.43		Q1	0.0105	-72.11
	Q1	0.0052	-21.68		K2	0.0076	-155.16

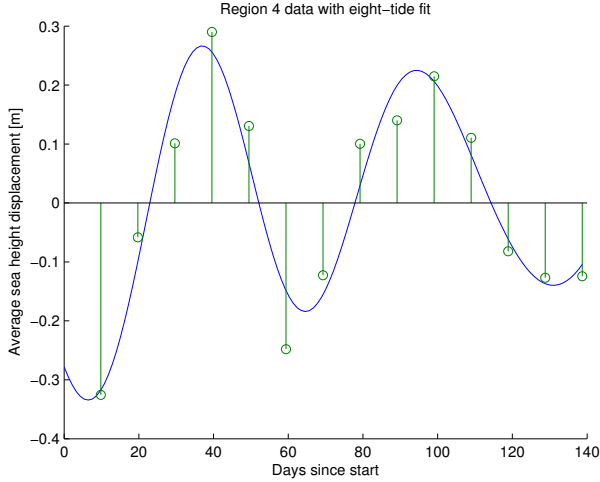


Fig. 4. Residual sea surface height for region 4 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.187 m.

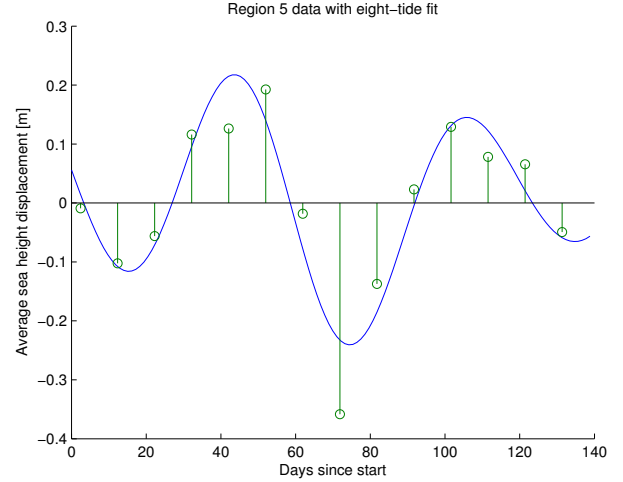


Fig. 5. Residual sea surface height for region 5 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.196 m.

effects, such as ocean currents, orbit errors, unmodeled or mis-modeled effects (such as in computing the range correction due to atmospheric loss), and other anomalous effects.

In all regions studied, the M2 tide is the strongest component. Additionally, in five of the six regions, M2, O1, and K1 are the three most dominant tides. S2, the chief solar component, is usually quite weaker than many other components for the six study regions. These results indicate that all of the eight tides selected for the paper are important to an understanding of ocean tides.

## V. CONCLUSION

Earth-orbiting altimeters such as Jason-2 provide important measurements of the sea surface level. Ocean tides are an important effect that must be estimated and ultimately removed from the residual data so further ocean topographical effects can be studied. In this paper, six small study regions from the open oceans have been selected and the average sea level for a pass within each region calculated. A time-series of each region is shown and the closest fit of eight tide components calculated. The largest components are usually due to M2, O1, and K1. For three of the six regions, M2 is at least an order of magnitude larger than any of the other components. For the other three, it is on the same order as O1 and K1. Clearly, tide

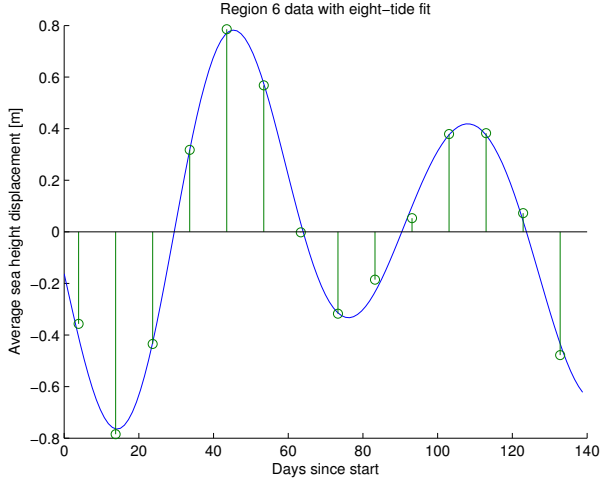


Fig. 6. Residual sea surface height for region 6 and the fit using the first eight tidal constituents. The  $\ell_2$  norm of the error is 0.109 m.

components are not spatially homogeneous—different locations are affected differently by the tidal components.

The results also show that while the eight-tide model fits the general trend of the data for the study regions, there is

still a small amount of variability that is unaccounted for. The simplistic tide model used is part of the reason behind the residual error. Other effects such as instrument noise or errors in other terms (Jason-2 orbit, atmospheric attenuation, mis-calibrated instruments) are also an issue, though these are expected to be small since Jason-2 is a third generation T/P-type design.

Jason-2 is a valuable tool to measuring sea surface levels and improving models of ocean tides. The Ku- and C-band altimeters permit all-weather measurements, regardless of cloud cover or solar illumination. A knowledge of ocean circulation and tide levels is important to understanding our planet.

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